

## AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

### Listing of Claims:

1           1. (Currently amended) A method for bounding the solution set of a  
2   system of linear equations  $\mathbf{Ax} = \mathbf{b}$ , wherein  $\mathbf{A}$  is an interval matrix and  $\mathbf{b}$  is an  
3   interval vector, the method comprising:  
4           receiving the system of linear equations  $\mathbf{Ax} = \mathbf{b}$ ;  
5           storing  $\mathbf{Ax} = \mathbf{b}$  in a memory in a computer system;  
6           preconditioning the set of linear equations  $\mathbf{Ax} = \mathbf{b}$  by multiplying ~~through~~  
7   both side of the linear equations by a matrix  $\mathbf{B}$  to produce a preconditioned set of  
8   linear equations  $\mathbf{BAx}=\mathbf{Bb}$ , wherein the set of linear equations is a representation  
9   of a global optimization problem;  
10          substituting  $\mathbf{M}_0 = \mathbf{BA}$  and  $\mathbf{r} = \mathbf{Bb}$  to produce  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ ;  
11          widening the matrix  $\mathbf{M}_0$  to produce a widened matrix  $\mathbf{M}$ , wherein the  
12   midpoints of the interval elements of  $\mathbf{M}$  form the identity matrix; and  
13          using  $\mathbf{M}$  and  $\mathbf{r}$  to compute ~~the hull a hull~~  $\mathbf{h}$  of the system  $\mathbf{Mx} = \mathbf{r}$ , which  
14   bounds the solution set of the system  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ ;  
15          wherein the interval operations involved in bounding the solution set are  
16   performed using a special-purpose interval arithmetic unit configured to perform  
17   interval arithmetic operations.

1           2. (Previously presented) The method of claim 1, wherein the method  
2   further comprises computing the matrix  $\mathbf{B}$  by:

3           computing an approximate center  $\mathbf{A}_C$  of the interval elements of matrix  $\mathbf{A}$ ;  
4    and  
5           forming  $\mathbf{B}$  by computing an approximate inverse of  $\mathbf{A}_C$ ,  $\mathbf{B} = (\mathbf{A}_C)^{-1}$ .

1           3 (Canceled).

1           4. (Previously presented) The method of claim 1, further comprising  
2    assuring that  $\sup(r_i) \geq 0$  by changing the sign of  $r_i$  ~~[[()]]~~ and  $x_i$  ~~[[()]]~~ if necessary,  
3    wherein  $r_i$  is an element of  $\mathbf{r}$ .

1           5. (Original) The method of claim 1, further comprising:  
2    determining if  $\mathbf{M}$  is regular; and  
3    using the Gauss-Seidel process for computing the hull  $\mathbf{h}$  if  $\mathbf{M}$  is not  
4    regular.

1           6. (Currently amended d) A computer-readable storage medium storing  
2    instructions that when executed by a computer cause the computer to perform a  
3    method for bounding the solution set of a system of linear equations  $\mathbf{Ax} = \mathbf{b}$ ,  
4    wherein  $\mathbf{A}$  is an interval matrix and  $\mathbf{b}$  is an interval vector, the method  
5    comprising:  
6           receiving the system of linear equations  $\mathbf{Ax} = \mathbf{b}$ ;  
7           storing  $\mathbf{Ax} = \mathbf{b}$  in a memory in a computer system;  
8           preconditioning the set of linear equations  $\mathbf{Ax} = \mathbf{b}$  by multiplying through  
9    both side of the linear equations by a matrix  $\mathbf{B}$  to produce a preconditioned set of  
10   linear equations  $\mathbf{BAx} = \mathbf{Bb}$ , wherein the set of linear equations is a representation  
11   of a global optimization problem;  
12           substituting  $\mathbf{M}_0 = \mathbf{BA}$  and  $\mathbf{r} = \mathbf{Bb}$  to produce  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ ;

13           widening the matrix  $\mathbf{M}_0$  to produce a widened matrix  $\mathbf{M}$ , wherein the  
14           midpoints of the interval elements of  $\mathbf{M}$  form the identity matrix; and  
15           using  $\mathbf{M}$  and  $\mathbf{r}$  to compute ~~the hull~~ a hull  $\mathbf{h}$  of the system  $\mathbf{M}\mathbf{x} = \mathbf{r}$ , which  
16           bounds the solution set of the system  $\mathbf{M}_0\mathbf{x} = \mathbf{r}_i$ ;  
17           wherein the interval operations involved in bounding the solution set are  
18           performed using a special-purpose interval arithmetic unit configured to perform  
19           interval arithmetic operations.

1           7. (Previously presented) The computer-readable storage medium of claim  
2           6, wherein the method further comprises computing the matrix  $\mathbf{B}$  by:  
3           computing an approximate center  $\mathbf{A}_C$  of the interval elements of matrix  $\mathbf{A}$ ;  
4           and  
5           forming  $\mathbf{B}$  by computing an approximate inverse of  $\mathbf{A}_C$ ,  $\mathbf{B} = (\mathbf{A}_C)^{-1}$ .

1           8. (Previously presented) The computer-readable storage medium of claim  
2           6, wherein using  $\mathbf{M}$  and  $\mathbf{r}$  to compute the hull  $\mathbf{h}$  involves:  
3           forming  $\mathbf{P}$  as an inverse of the left endpoint of  $\mathbf{M}$ ;  
4           forming  $c_i = 1/(2P_{ii} - 1)$  for  $i = 1, \dots, n$ ;  
5           forming  $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$ , wherein  $e_i^T$  is a unit vector in  
6           which the  $i$ -th element is 1 and other elements are 0, and wherein  $r_i$  is an element  
7           of  $\mathbf{r}$ ;  
8           setting  $\inf(h_i) = c_i z_i$  if  $z_i > 0$ ;  
9           setting  $\inf(h_i) = z_i$  if  $z_i \leq 0$ ; and  
10          setting  $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$ .

1           9. (Previously presented) The computer-readable storage medium of claim  
2           6, wherein the method further comprises assuring that  $\sup(r_i) \geq 0$  by changing the  
3           sign of  $r_i$   $[[[]]$  and  $x_i[[]]$  if necessary, wherein  $r_i$  is an element of  $\mathbf{r}$ .

1           10. (Original) The computer-readable storage medium of claim 6, wherein  
2 the method further comprises:  
3           determining if  $\mathbf{M}$  is regular; and  
4           using the Gauss-Seidel process for computing the hull  $\mathbf{h}$  if  $\mathbf{M}$  is not  
5 regular.

1           11. (Currently amended) An apparatus that bounds the solution set of a  
2 system of linear equations  $\mathbf{Ax} = \mathbf{b}$ , wherein  $\mathbf{A}$  is an interval matrix and  $\mathbf{b}$  is an  
3 interval vector, comprising:  
4           a receiving mechanism configured to receive the system of linear  
5 equations  $\mathbf{Ax} = \mathbf{b}$ ;  
6           a special purpose arithmetic unit configured to perform interval  
7 computations;  
8           a storing mechanism configured to store  $\mathbf{Ax} = \mathbf{b}$  in a memory in a  
9 computer system;  
10          a preconditioning mechanism within the special purpose arithmetic unit  
11 that is configured to precondition the set of linear equations  $\mathbf{Ax} = \mathbf{b}$  by  
12 ~~multiplying through both side of the linear equations~~ by a matrix  $\mathbf{B}$  to produce a  
13 preconditioned set of linear equations  $\mathbf{BAx} = \mathbf{Bb}$ , wherein the set of linear  
14 equations is a representation of a global optimization problem;  
15          a substituting mechanism within the special purpose arithmetic unit that is  
16 configured to substitute  $\mathbf{M}_0 = \mathbf{BA}$  and  $\mathbf{r} = \mathbf{Bb}$  to produce  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ ;  
17          a widening mechanism within the special purpose arithmetic unit that is  
18 configured to widen the matrix  $\mathbf{M}_0$  to produce a widened matrix  $\mathbf{M}$ , wherein the  
19 midpoints of the interval elements of  $\mathbf{M}$  form the identity matrix; and  
20          a hull computing mechanism within the special purpose arithmetic unit  
21 that is configured to use  $\mathbf{M}$  and  $\mathbf{r}$  to compute the hull a hull  $\mathbf{h}$  of the system  
22  $\mathbf{Mx} = \mathbf{r}$ , which bounds the solution set of the system  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ ;

23        wherein the interval operations involved in bounding the solution set are  
24        performed using the special-purpose interval arithmetic unit.

1            12. (Previously presented) The apparatus of claim 11, wherein the  
2        preconditioning mechanism is configured to:  
3            compute an approximate center  $\mathbf{A}_C$  of the interval elements of matrix  $\mathbf{A}$ ;  
4        and to  
5            form  $\mathbf{B}$  by computing an approximate inverse of  $\mathbf{A}_C$ ,  $\mathbf{B} = (\mathbf{A}_C)^{-1}$ .

1            13. (Previously presented) The apparatus of claim 11, wherein the hull  
2        computing mechanism is configured to:  
3            form  $\mathbf{P}$  as an inverse of the left endpoint of  $\mathbf{M}$ ;  
4            form  $c_i = 1/(2P_{ii} - 1)$  for  $i = 1, \dots, n$ ;  
5            form  $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$ , wherein  $e_i^T$  is a unit vector in  
6        which the  $i$ -th element is 1 and other elements are 0, and wherein  $r_i$  is an element  
7        of  $\mathbf{r}$ ;  
8            set  $\inf(h_i) = c_i z_i$  if  $z_i > 0$ ;  
9            set  $\inf(h_i) = z_i$  if  $z_i \leq 0$ ; and to  
10          set  $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$ .

1            14. (Previously presented) The apparatus of claim 11, wherein the  
2        preconditioning mechanism is configured to assure that  $\sup(r_i) \geq 0$  by changing  
3        the sign of  $r_i$   $[[()]]$  and  $x_i[[]]$  if necessary, wherein  $r_i$  is an element of  $\mathbf{r}$ .

1            15. (Original) The apparatus of claim 11, wherein the preconditioning  
2        mechanism is configured to:  
3            determine if  $\mathbf{M}$  is regular; and to  
4            terminate the process of computing the hull  $\mathbf{h}$  if  $\mathbf{M}$  is not regular.

1           16. (Currently amended) A method for bounding the solution set of a  
2   | system of linear equations  $\mathbf{Ax} = \mathbf{b}$  by multiplying ~~through both side of the linear~~  
3   | equations by the matrix  $\mathbf{B}$  to produce a preconditioned set of linear equations  
4   |  $\mathbf{BAx} = \mathbf{Bb}$ , wherein the set of linear equations is a representation of a global  
5   | optimization problem, the method comprising:  
6           receiving the system of linear equations  $\mathbf{Ax} = \mathbf{b}$ ;  
7           storing  $\mathbf{Ax} = \mathbf{b}$  in a memory in a computer system;  
8           substituting  $\mathbf{M}_0 = \mathbf{BA}$  and  $\mathbf{r} = \mathbf{Bb}$  producing  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ ;  
9           assuring that  $\sup(r_i) \geq 0$  by changing the sign of  $r_i$  (and  $x_i$ ) if necessary;  
10          widening the matrix  $\mathbf{M}_0$  to produce a widened matrix  $\mathbf{M}$ , wherein the  
11   | midpoints of the interval elements of  $\mathbf{M}$  form the identity matrix; and  
12   |          using  $\mathbf{M}$  and  $\mathbf{r}$  to compute the ~~hull~~ a hull  $\mathbf{h}$  of the system  $\mathbf{Mx} = \mathbf{r}$ , which  
13   | bounds the solution set of the system  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$  by,  
14                  forming  $\mathbf{P}$  as an inverse of the left endpoint of  $\mathbf{M}$ ,  
15                          forming  $c_i = 1/(2P_{ii} - 1)$  for  $i = 1, \dots, n$ ,  
16                          forming  $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$ ,  
17                          wherein  $e_i^T$  is a unit vector in which the  $i$ -th element is 1  
18                          and other elements are 0, and wherein  $r_i$  is an element of  $\mathbf{r}$ ,  
19                          setting  $\inf(h_i) = c_i z_i$  if  $z_i > 0$ ,  
20                          setting  $\inf(h_i) = z_i$  if  $z_i \leq 0$ , and  
21                          setting  $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$ ;  
22          wherein the interval operations involved in bounding the solution set are  
23   | performed using a special-purpose interval arithmetic unit configured to perform  
24   | interval arithmetic operations.

1           17. (Original) The method of claim 16, further comprising:  
2           determining if  $\mathbf{M}$  is regular; and

3           using the Gauss-Seidel process for computing the hull **h** if **M** is not  
4   regular.

1           18. (Previously presented) The method of claim 16, wherein the method  
2   further comprises computing the matrix **B** by:  
3           computing an approximate center **A<sub>C</sub>** of the interval elements of matrix **A**;  
4   and  
5           forming **B** by computing an approximate inverse of **A<sub>C</sub>**, **B** = (**A<sub>C</sub>**)<sup>-1</sup>.